

The entropy of elliptical galaxies in Coma: a clue for a distance indicator[★]

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ABSTRACT

We have fitted the surface brightness of a sample of 79 elliptical galaxies pertaining to the Coma cluster of galaxies using the Sérsic profile. This model is defined through three primary parameters: scale length (a), intensity (Σ_0), and a shape parameter (ν); physical and astrophysical quantities may be computed from these parameters. We show that correlations are stronger among primary parameters than the classical astrophysical ones. In particular, the galaxies follow a high correlation in ν and a parameters. We show that the ν and a correlation satisfies a constant specific entropy condition. We propose to use this entropy relation as distance indicator for clusters.

Key words: galaxies: clusters: Coma – galaxies: distances – galaxies: fundamental parameters – distance scale

1 INTRODUCTION

The photometrical properties of elliptical galaxies have already been extensively analysed, with special attention paid to the entire range of sizes and luminosities, from giant to dwarf ellipticals. Profile laws are in general defined by various parameters, for example, scale length (a), intensity (Σ_0), and one or more parameters specifying the shape (structure parameters). These parameters can be categorized as *basic* or *primary*, in the sense that they come out naturally from a mathematical definition.

The de Vaucouleurs profile has no structure parameter; for a long time it appeared to be in very good agreement with observations, although hints of systematic deviations were observed; those deviations appear similar for galaxies with the same luminosities (Michard 1985, Schombert J.M. 1986). New observations, especially high resolution data from the HST (e.g. Ferrarese et al. (1994)), have definitively shown that the de Vaucouleurs' profile – rather too inflexible – does not allow a convenient description of the actual structure of all elliptical galaxies.

Some attempts have been made to obtain more general

surface brightness laws. Among many possibilities, Sérsic's recipe (1968),

$$\Sigma(r) = \Sigma_0 \exp(-u^\nu), \quad u = r/a, \quad (1)$$

a generalization of the de Vaucouleurs-law, seems to be better suited to describe elliptical brightness profiles (e.g. Ciotti (1991), Caon et al. (1993), Graham et al. (1996), Courteau et al. (1996)).

In this letter we show the results of the photometrical analysis of a sample of elliptical galaxies in the Coma cluster using a Sérsic-model (or a ν model). From the basic parameters [a , Σ_0 , ν], it is possible to build physical and astrophysical quantities (effective radius, effective luminosity, total magnitude, energy, entropy, etc...). All these quantities are in fact a combination of primary parameters; we therefore propose to use primary parameters instead of the classical 'astrophysical' quantities in order to obtain better defined correlations.

Any correlation between a dimensionless parameter and a distance-dependent one has traditionally led to the definition of a distance indicator; for instance, the Cepheid luminosity–period relation (Luminosity/Period), the Tully-Fisher relation (Luminosity/Rotation), and the Faber-Jackson relation (Luminosity/Velocity dispersion) are widely used. Recently, Young & Currie (1994) have pointed out a correlation between the (distance-dependent) intrinsic

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luminosity and the (dimensionless) shape parameter of the Sérsic-law for dwarf ellipticals.

A theoretical understanding of the physical underlying processes is crucial to assure the validity of a correlation between the parameters describing an astronomical object. Here, we show that elliptical galaxies are located in the locus defined by constant specific entropy s , in the primary parameters plane $[\nu, a]$. The fact that the elliptical galaxies in Comamainly have the same specific entropy should somewhat reflect the intrinsic properties of self-gravitating systems formed through violent relaxation processes. We will discuss whether the entropy relation connecting ν and a may be used as a distance indicator.

2 THE SÉRSIC-MODEL

2.1 Scale and structure parameters

Surface brightness functions generally used to describe elliptical galaxies can be written as:

$$\Sigma = \Sigma_s \bar{F}(|\vec{\alpha}|; (u)), \quad u = (r/a) \quad (2)$$

where \bar{F} does not depend on the normalization factor, Σ_s . Σ_s is a characteristic intensity and a is a characteristic scale. Both are scaling parameters, while the set $\vec{\alpha}$, contains the structural parameters which account for the specific shape of the surface brightness profile. The so-called β -model, γ -model, or more generally the (α, β, γ) -model (Zhao 1996), are known examples.

From both scaling and structural parameters which are *primary* in the sense of being directly obtained from the fit, total luminosity, effective radius, effective magnitude, etc... may be calculated. Since they result from a combination of the primary parameters, they can be considered as *secondary quantities*.

2.2 The Sérsic distribution

To proceed with the fit we have decided to use a general brightness profile pertaining to the family described by Eq. (2), namely the Sérsic model. The fitting function, that we will refer as the ν -model (due to the analogy with the β -model) is given by Eq. (1).

The relation between ν and n (which is often used) is $n = 1/\nu$. For $\nu = 0.25$ (i.e., $n = 4$) the de Vaucouleurs-model is recovered. We note that the de Vaucouleurs-model has no structural parameter, implying that any galaxy profile can be deduced from a unique galaxy model by scaling (homology property).

2.3 Total and effective quantities

To calculate the total luminosity and ‘effective’ quantities (i.e. quantities related to the ‘effective’ radius R_e) we have used the classical definitions and, after straightforward calculations, we find:

- for the total luminosity, L_{tot} :

$$L_{\text{tot}} = \Sigma_0 a^2 L^*(\nu) \quad (3)$$

with:

$$L^*(\nu) = 2\pi \frac{1}{\nu} \Gamma\left(\frac{2}{\nu}\right) \quad (4)$$

where $\Gamma(x)$ is the Gamma function. $L^*(\nu)$ depends only on the structure parameter ν . The total magnitude is $m_{\text{tot}} = -2.5 \log L_{\text{tot}} + \text{cte}$. We have approximated the magnitude $m^*(\nu) = -2.5 \log L^*(\nu)$ by the following expression:

$$m^*(\nu) = 2.199 - 1.246 \nu - \frac{2.720}{\nu} - \frac{0.2195}{\nu^2} \quad (5)$$

which is accurate to 3% for $0.1 \leq \nu \leq 1.0$.

- for the effective Radius: $R_e = a U_e$, where $U_e(\nu)$ depends only on ν ; we have approximated it by an analytic expression:

$$\ln U_e(\nu) = \frac{1}{\nu} (0.5434 - 1.069 \ln \nu) \quad (6)$$

- for the effective brightness, μ_e :

$$\mu_e = -2.5 \log(\Sigma_0) + 1.0857 U_e^\nu(\nu) \quad (7)$$

As it can be seen, μ_e is the sum of two terms; the first one is a function of Σ_0 and the second one only depends on ν .

- For the mean effective brightness, $\langle \mu \rangle_e$:

$$\langle \mu \rangle_e = 2.5 \log 2\pi - 2.5 \log \Sigma_0 + 5 \log U_e(\nu) + m^*(\nu) \quad (8)$$

$\langle \mu \rangle_e$ is the sum of 4 terms, the first being a function of Σ_0 while the second and third depend only on ν .

Therefore, the photometrical properties of elliptical galaxies can be described in terms of a combination of primary parameters. That is to say, we may transform the triplet $[\Sigma_0, a, \nu]$ into the triplet $[L_{\text{tot}}(m_{\text{tot}}), R_e, \nu]$. Correlations between the secondary parameters $[L_{\text{tot}}(m_{\text{tot}}), R_e, \nu]$ should therefore show larger dispersions than those corresponding to $[\Sigma_0, a, \nu]$. Notice that any strong correlation between either Σ_0 or a , and ν should provide a distance indicator.

3 ANALYSIS OF THE DATA AND CORRELATIONS

3.1 The data

The sample of elliptical galaxies has been selected from the catalogue by Lobo et al. (1997), based on V-band CCD images covering a region of $30' \times 21'.6$ centered on the position $\alpha = 12^h 59^m 42^s 71$, $\delta = +27^\circ 58' 14'' 2$, observed at the 3.6 m Canada–France–Hawaii Telescope in 1993. We have selected 79 galaxies, which cover the range $[13-17.8]$ in V-magnitudes and $[0.6-20 \text{ kpc}]$ in effective radius. We assume a distance for Coma of $137 h_{50}^{-1} \text{ Mpc}$ (Colless & Dunn 1996). A full description of the sample will be given elsewhere (Gerbal et al. 1997).

3.2 The fitting method

We have fitted the integrated flux (the luminosity growth curve) as a function of the surface. We decided to use such a fit since the irregularities of the galaxy are smoothed out. While fitting the light profile is very sensitive to the outer parts, fitting the integrated flux gives a better representation of the global light distribution (Prugniel & Simien (1996)). Moreover, in this way we can take into account the ellipticity of the galaxies.

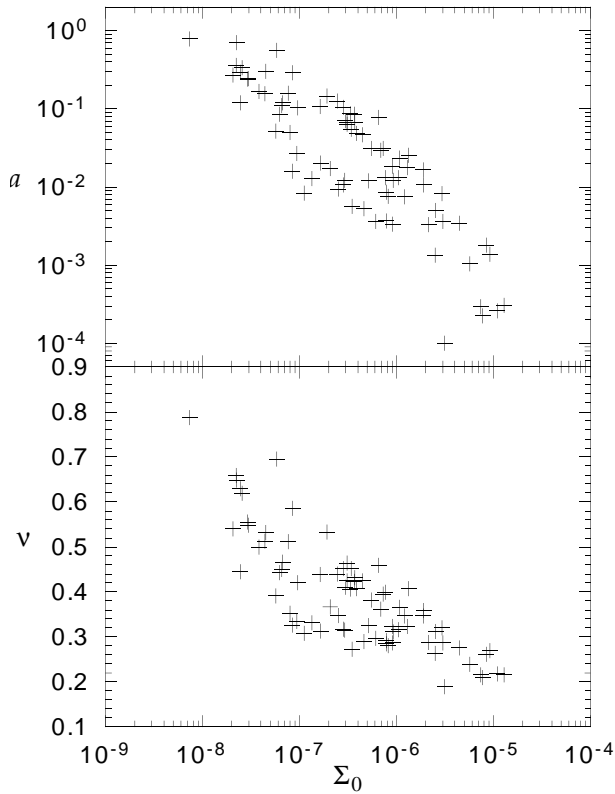


Figure 1. Distribution of the primary parameters a , Σ_0 (top), and ν , Σ_0 (bottom). ν is a dimensionless parameter, a is expressed in $\text{kpc } h_{50}^{-1}$, and Σ_0 in $\text{erg arcsec}^{-2} \text{ s}^{-1}$

We therefore write equation (1) in an integrated form:

$$L_{\text{tot}}(\varepsilon) = \frac{2b}{\nu} \Sigma_0 \gamma\left(\frac{2}{\nu}, \left(\frac{\varepsilon}{b}\right)^{\nu/2}\right), \quad (9)$$

where $\gamma(c, x)$ is the incomplete gamma function, b is related to the length scale a by $b = \pi a^2$, and ε is the surface corresponding to the region for which the flux is higher than a given level Σ (cf. Eq.(1)). The corresponding mean radius is then defined as $r = (\pi\varepsilon)^{1/2}$. $L_{\text{tot}}(\varepsilon)$ is the total flux within a given ε .

The sky level was determined in a similar way as Jørgensen & Franx (1994). In order to limit the influence of the seeing ($\text{FWHM} \approx 0.9''$) the data points are taken from $3.0''$ outwards, out to a surface brightness $\mu_V = 24.0$ (i.e. 3σ signal above the background sky noise). The fitting process has been done under Interactive Data Language (IDL) supplying data values with their $1\text{-}\sigma$ errors. These are very small ($\sim 1\%$, 3% , and 2% for ν , a , Σ_0 , respectively) and are correlated with each other as in the de Vaucouleurs or β -model case. A more detailed description of the fitting procedure will be given elsewhere (Gerbal et al. 1997), together with the resulting values for primary and secondary parameters.

3.3 Correlations between parameters

In Fig. 1 and Fig. 2 we show the relations between the 3 primary parameters: $[\nu, \Sigma_0]$, $[a, \Sigma_0]$, and $[a, \nu]$.

In order to quantify the correlations in Figs. 1 and 2, we have performed the Spearman and Kendall rank correla-

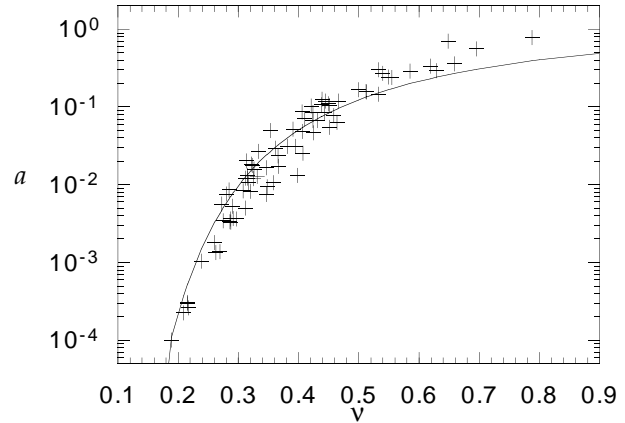


Figure 2. Distribution of the primary parameters a , ν . The units are as in Fig. 1. The solid line represents a constant specific entropy track (cf. section 4.1).

Table 1. Correlation between the parameters of the Sérsic-law. Both ρ and τ are defined in the interval $[-1, 1]$, the 0 value meaning no correlation. Higher absolute values of z indicate a greater significance of the correlations

Parameters	ρ -Spearman	z -value	τ -Kendall	z -value
ν - a	0.97	8.53	0.85	11.06
ν - Σ_0	-0.80	-7.08	-0.62	-8.07
a - Σ_0	-0.85	-7.52	-0.68	-8.88
ν - R_e	-0.54	-4.73	-0.38	-4.91
ν - m_{tot}	0.48	4.23	0.34	4.42

tion (non-parametric) tests (Siegel & Castellan 1989). The results are presented in Table 1.

It can be seen that there are correlations between all primary parameters. However the correlations related to the normalization factor Σ_0 are less pronounced, while a and ν are strongly correlated. The greater a , the greater ν , i.e., the greater the characteristic length, the steeper the brightness profile.

In Fig. 3 we show the correlations between the secondary parameters $[\nu, R_e]$ and $[\nu, m_{\text{tot}}]$. The secondary parameters effective radius, R_e and total magnitude, m_{tot} , are clearly much less strongly correlated with ν .

This result could be explained in terms of the noisy combination of the primary parameters to obtain the second ones and, to a certain extent, the role played by Σ_0 (see eq. 3), which is not so well correlated to the two other primary parameters.

4 IMPLICATIONS OF THE RELATION $[a, \nu]$

4.1 A physical interpretation

As noted by Graham et al. (1996), the physical origin of the $[a, \nu]$ correlation is not understood up to now. We will show that this relationship has a simple physical interpretation in terms of gravo-thermodynamics. Notice that this kind of relation exists for a large family of objects – from the nuclei of spiral galaxies (Courteau et al. 1996) to Brighter Cluster

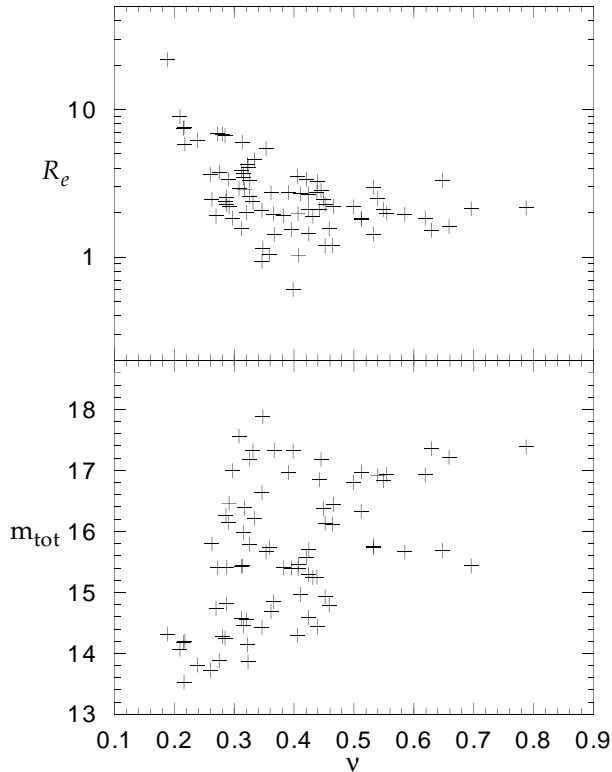


Figure 3. Correlations between ν and two secondary parameters: R_e (in $\text{kpc } h_{50}^{-1}$, top) and m_{tot} (in magnitude, bottom).

galaxies, and even for the X-ray emitting gas in galaxy clusters (Durret et al. 1994). Therefore, the physical explanation should be only related to a and ν and to a small extent to Σ_0 .

Gravitational dynamical systems in quasi-equilibrium are in a state of quasi-stationary entropy. We study here the variation of the entropy with the parameters a and ν of the Sérsic-law. We use a modified deprojection of the de Vaucouleurs law derived by Mellier and Mathez (1987) to represent the 3D-density:

$$\rho(r) = \rho_0 \left(\frac{r}{a} \right)^{-p} \exp \left(- \left(\frac{r}{a} \right)^\nu \right), \quad (10)$$

where ρ_0 is the normalization parameter. For any ν_i there is one p_i so that the ν_i -model should be recovered by projecting $\rho(r)$ in 2-D. When $\nu = 0.25$ then $p = -0.855$. In order to recover the Sérsic-law from the projection of Eq. (10) we have derived, following Mellier & Mathez, the relation:

$$p(\nu) = 0.9976 - 0.5772\nu + 0.03243\nu^2, \quad (11)$$

which is accurate to better than 0.1% in the range $0.1 \leq \nu \leq 1.0$.

Assuming spherical symmetry, hydrostatic equilibrium, and an isotropic velocity tensor, it is straightforward to compute the total mass, M_{tot} (assuming a constant mass-to-light ratio), potential energy, U , and pressure profile, $P(r)$. Finally, the entropy (\mathcal{S}), and the specific entropy (s) (following White & Narayan (1987), assuming an ideal gas), are given by:

$$s = \frac{\mathcal{S}}{M_{\text{tot}}} = \frac{1}{M_{\text{tot}}} \int_V \ln(\rho^{-5/2} P^{3/2}) \rho \, dV, \quad (12)$$

the integral is over all the volume V of the galaxy.

In Fig. 2 we have plotted the specific entropy as a function of a and ν . As can be expected, the entropy of a self-gravitational system of unlimited extent has no maximum. With the present calculation it increases with ν and with decreasing a . Superimposed to the specific entropy curve (isentropic track) are the observed values of a and ν from the fits; one can note that they closely follow one another.

This result suggests that there should exist some intrinsic relation between a and ν , the structural parameters of the Sérsic-law. However, in order to be able to obtain a maximum entropy in a dynamical system, one needs to impose more constraints, in addition to Eq. (10). The isentropic track occupied by the observed values of a and ν may then be understood as a maximum entropy class of some more restricted distribution.

4.2 Towards a distance indicator

The ν -model, contrary to the de Vaucouleurs-law, does not allow to deduce any brightness profile just by scaling; the homology property of the de Vaucouleurs-law is lost. Instead, the shape property (ν) is related to the scale (a) by the relation of constant entropy (12), $s = s_0$. Since a depends on the distance whereas ν does not, their correlation can be used as a distance indicator.

All the galaxies used for the fit belong to the Coma cluster, so they are at the same distance (the cluster membership has been determined with the redshift, see Biviano et al. 1995), and thus provide us with a calibration for the distance indicator based on the a - ν relation. If the entropy relationship were ‘universal’, i.e., if it were the same for different clusters, it could be used as a robust distance indicator. To any observed $\nu_{(k)}$ of any galaxy g_k in the Universe, would then correspond an a_k (in angle units) which may be compared to the corresponding value obtained from the entropy-relation (Fig. 2), allowing to derive the distance to g_k .

One could wonder if the a - ν relation is weakened by the correlation between the fitting parameters for a given galaxy. Since the isentropic track is predicted by theory, we actually believe that the a - ν relation is robust.

The dispersion of the residue distribution of $\Delta = (a_{\text{calc}} - a)$ is $\simeq 0.1$ leading to a $1-\sigma$ error of 10% of the residue. The observed dispersion of Δ could be due to some difficulties in the fitting process, or due to physical reasons. In fact, it seems reasonable to imagine that a galaxy which has undergone some energy and entropy exchanges with other galaxies (merging, tidal effects, etc. . .) could not verify the entropy relation.

Notice that the ν -profile is a 2-D brightness profile, whereas a 3-D density profile is needed to calculate the specific entropy. We have used a constant mass-to-light ratio to transform from one to the other. Therefore, those galaxies being more luminous than normal ones (for instance a galaxy hosting a starburst), will be out of the theoretical relationship.

5 CONCLUSIONS

We have given a physical interpretation for the observed relation between two of the *primary* parameters of the ν -profile: the values of the a and ν parameters of Coma cluster galaxies approximately have a constant specific entropy. The dispersion around the best fit is small, therefore allowing the use of this relation as a distance indicator (relative to the Coma cluster).

We must stress that both the observational and theoretical evidences found in this work have to be verified. On the one hand, it is necessary to quantify the universality (i.e. in other clusters) of the (a, ν) correlation found for the galaxies in the Coma cluster. On the other hand, the validity of the assumptions made to derive the theoretical entropy relationship (constant M/L , ideal gas entropy) must be confirmed.

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